Note on the Time Reversal Asymmetry of Equations of Motion[1]

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Abstract

Rohrlich's recent claim that the equation of motion for a point charge be symmetric under time reversal is shown to be the result of an unusual definition. The equation of motion for a charged sphere of finite size, which in contrast is claimed to be asymmetric because of the finite propagation time of its (retarded) self-forces, is shown to possess the same asymmetry (or the same symmetry, depending on the definition) as that for a point charge. Similar arguments apply to other effective equations of motion (such as those describing friction or decoherence).

Key words: radiation reaction, Lorentz-Dirac equation, irreversibility, decoherence.

[1] Unfortunately, several misleading errors have been edited into the printed version of this contribution (Foundations of Physics Letters, **12**, 193), including its title. Note that this e-print represents the correct version.

Rohrlich argued recently[1] that the Lorentz-(Abraham)-Dirac (LAD) equation for a point charge moving with four-velocity $v^\mu(\tau)$ along its world line,

$$m\dot{v}^\mu = \frac{2e^2}{3}(\ddot{v}^\mu - v^\mu \dot{v}^\nu \dot{v}_\nu) + F_{\rm in}^{\mu\nu} v_\nu \quad , \tag{1}$$

be invariant under time reversal. Here, m is the renormalized (physical) mass, while $F_{\rm in}^{\mu\nu}$ is an external Maxwell field. Rohrlich's claim may appear surprising, since this equation contains terms proportional to the velocity and to the first time derivative of the acceleration, both of which change sign under time reversal — similar to the terms responsible for friction in the equation of motion for a mass point. These two terms describe the *loss* of energy that must balance the *emission* of radiation (Dirac's radiation reaction) and the ill-defined self-acceleration that leads to exponentially increasing velocity. Both terms arise from the presumed retardation (the absence of the advanced fields of the point charge).

It may appear even more surprising when Rohrlich also insists that the Caldirola-Yaghjian (CY) equation[2, 3] for a charged sphere of radius a,

$$m_0\dot{v}^\mu(\tau) = \frac{2e^2}{3a}\frac{v^\mu(\tau-2a) + v^\mu(\tau)v^\nu(\tau)v_\nu(\tau-2a)}{2a} + F_{\rm in}^{\mu\nu} v_\nu(\tau) \quad , \tag{2}$$

with bare mass $m_0 = m - 2e^2/3a$, be asymmetric, even though the former equation can be obtained from the latter in the limit $a \to 0$ under a time-symmetric though divergent mass renormalization. Apparently, the idea underlying these claims is that only the retarded self-forces within the sphere create an asymmetry, which must then disappear in the point limit. I will now argue that this picture is wrong.

What Rohrlich does in fact prove in the first part of his paper (and also in his book[4]) is the invariance of the LAD equation under time reversal provided this is defined to include a simultaneous interchange of incoming and outgoing fields. However, this symmetry of the *global* situation merely reflects that of the complete theory; it does not represent the behavior under time reversal of a point charge that would always be allowed freely to create its *retarded* radiation. Although the claim is then formally correct, it is based on a very unusual and misleading definition of time reversal for an equation of motion. This becomes obvious when the definition is correspondingly applied to friction, which would also be time reversal invariant if the second law were simultaneously reversed (that is, if dissipated heat were replaced with heat focussing on the mass point instead of the retarded fields being

replaced with advanced ones in the case of the charge). In the same sense, the Lorentz force is symmetric under time reversal if its magnetic fields are simultaneously reflected in space. These three situations differ only in the complexity of their “environments”, and hence in the *practical* difficulties of time-reversing them for this Loschmidt-type argument. Their symmetry is thus trivial (as Rohrlich correctly points out for the LAD equation), but essentially of mere theoretical importance in the first two cases.

Rohrlich’s symmetry of the LAD equation is based on the general equivalence of the two familiar representations of an arbitrary field, *viz.* either as a sum of incoming and retarded fields (of the sources in the considered spacetime volume) or of outgoing and advanced fields. Their essential difference is that it is easy to control incoming fields, but hard to manipulate outgoing ones. Moreover, fields often vanish before sources are turned on, while they do not after the sources are turned off. This familiar *fact* is a consequence of the general presence of absorbers (such as laboratory walls) with their thermodynamical arrow of time[5].

This equivalence of different representations may also be applied to the nonsingular case of a charged sphere of finite size. One may either add its retarded field to a given incoming field or the advanced field to the outgoing one in order to satisfy the Maxwell equations. The first choice leads to the CY equation (2), while the second one would mean that the retarded arguments $\tau - 2a$ in (2) have to be replaced with $\tau + 2a$ because of the advanced self-forces that are now *required* to act within the charged sphere. Instead of this replacement, Rohrlich leaves this expression in its retarded form in spite of his definition of time reversal, since “advanced interactions are never observed” and “should not be possible”. However, the interchange of incoming and outgoing fields required in his definition of time reversal should then neither be possible. (In practice, one would have to prepare the complete though time-reversed retarded field coherently as an incoming field.) Retardation or advancement are here a consequence of the chosen representation — not of the empirical situation in our world. When formally fixing outgoing fields, one has to use advanced fields of the considered sources everywhere. Advanced external fields would be inconsistent in conjunction with retarded internal fields.

One may then consider the limit $a \to 0$ in the CY equation by using the Taylor expansion $v^\mu(\tau \mp 2a) = v^\mu(\tau) \mp 2a\dot{v}^\mu(\tau) + 2a^2\ddot{v}^\mu(\tau) + \ldots$ and the condition $v^\mu v_\mu = -1$ together with its time derivatives (that is, $v^\mu \dot{v}_\mu = 0$ and $v^\mu \ddot{v}_\mu = -\dot{v}^\mu \dot{v}_\mu$). The first order of this Taylor expansion gives the mass renormalization $\Delta m = 3e^2/2a$, while the second one leads precisely to the

LAD equation, with signs of the retardation differing in the two cases. The limit $a \to +0$ is thus nontrivially different from the limit $a \to -0$. (This is related to the well known fact that master equations are trivial in first order of the interaction.) Therefore, the LAD equation and the CY equation possess the *same asymmetry* under time reversal in the usual sense, and the *same symmetry* in the sense of Rohrlich.

Equivalent concepts of time reversal are valid for other equations of motion[6]. They can also be applied to the master equation of a mass point described quantum mechanically under the effect of decoherence[7, 8],

$$i\frac{\partial \rho(x,x',t)}{\partial t} = \frac{1}{2m}\left(\frac{\partial^2}{\partial x'^2} - \frac{\partial^2}{\partial x^2}\right)\rho - i\lambda(x-x')^2\rho \quad , \tag{3}$$

where ρ is the density matrix. This equation is again asymmetric in the usual sense, since it reflects the *formation of retarded entanglement* (quantum correlations). *All* these equations of motion are asymmetric if regarded by themselves, since their dynamical objects (such as mass points) are strongly coupled to a time-directed environment, while they would be symmetric if they were time-reversed together with their environment.

In the case of decoherence, time reversal of the environment would require recoherence, that is, the conspiratorial presence of previously unobserved but presicely matching Everett components ("parallel worlds"). However, a fundamental collapse of the wave function (if it existed as a dynamical law) would not even theoretically allow the environment to be time-reversed. While all these situations may reflect the same master arrow of time[5] (that is, the same cosmic initial condition), it remains open whether there is a boundary somewhere that separates reversible from irreversible physics in a fundamental (law-like) way. Most physicists appear ready to accept such a boundary in conjunction with equation (3), that is, for quantum mechanical measurements or related "probabilistic quantum events" — wherever the precise boundary between this collapse of the wave function and the realm of the Schrödinger equation may be located.

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